

MATH 123 SAMPLE GATEWAY EXAM

This is a sample of the gateway exam that will be given to all students enrolled in MATH 123 (Calculus I) on the second day of class. The sample exam is quite extensive whereas the actual exam will have 10 to 15 problems. Students will have 30 minutes to complete the actual exam and **NO CALCULATORS** are allowed for the exam.

1. Given $f(x) = \sqrt{x+2}$, evaluate and simplify $\frac{f(x+h) - f(x)}{h}$.

2. Evaluate

$$\sin\left(\frac{2\pi}{3}\right) = \underline{\hspace{2cm}}$$

$$\cos\left(\frac{2\pi}{3}\right) = \underline{\hspace{2cm}}$$

$$\tan\left(\frac{2\pi}{3}\right) = \underline{\hspace{2cm}}$$

3. Find all solutions for $2\sin^2 x - \sin x = 0$ for $0 \leq x \leq 2\pi$.

4. Farmer Jed wants to build a pigpen with length 2 feet longer than the width and a total area of 255 square feet. How much fence will Farmer Jed need for his pigpen?

5. Write the equation of the line (**in point-slope form**) that passes through the points $(-3, 4)$ and $(1, -2)$.

6. Solve the following equations for x .

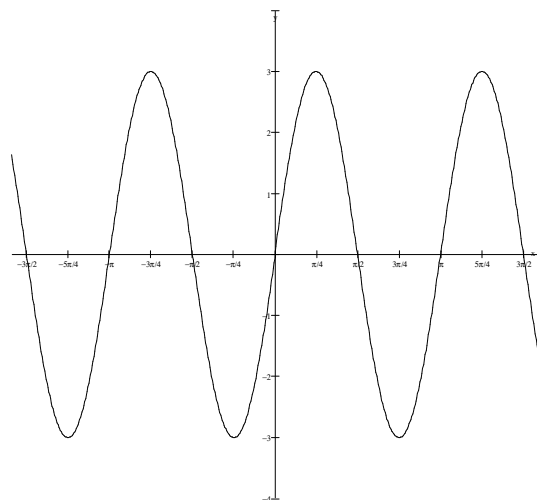
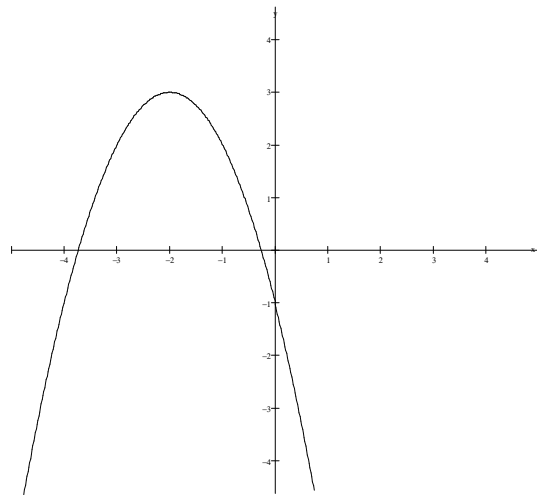
(a) $e^x = 2$

(b) $\ln x - \ln 3 = 2$

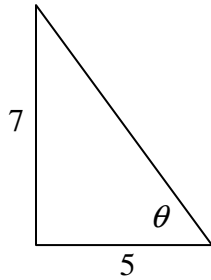
7. (a) Find a , h , and k so that $f(x) = a(x-h)^2 + k$ represents the graph to the right.

(b) Find A , B , C , and D so that

$f(x) = A \sin(B(x-C)) + D$ represents the graph below.



8. Given the triangle below, evaluate the following.



$$\sin \theta = \underline{\hspace{2cm}}$$

$$\cos \theta = \underline{\hspace{2cm}}$$

$$\tan \theta = \underline{\hspace{2cm}}$$

9. Factor the following completely.

(a) $\frac{(t+2)^{-2}(-2t^2-27)+t^2(t+2)^{-1}}{(t+2)^3}$

(b) $x^{-1/2}(2x^2-5x-28)-x^{1/2}(x-2)$

$$\frac{1}{x} - \frac{1}{x-2}$$

10. Simplify $\frac{x-2}{x-2}$.

11. Find the domain of $f(x) = \frac{1}{\sqrt{x^2-x-6}}$.

12. Solve $\frac{1}{x} + \frac{2}{x-1} = 7$ for x .

13. Simplify the expressions and eliminate any negative exponent(s).

(a) $\frac{(xy^2)^2(x^3y)^{-4}}{x^2y^{-4}}$

(b) $(16x^8)^{1/2}(27x^9)^{-1/3}$

14. If a ball is thrown vertically upward from the ground, its height in feet after t seconds is given by the function $s(t) = 80t - 16t^2$.

(a) What is the maximum height of the ball?

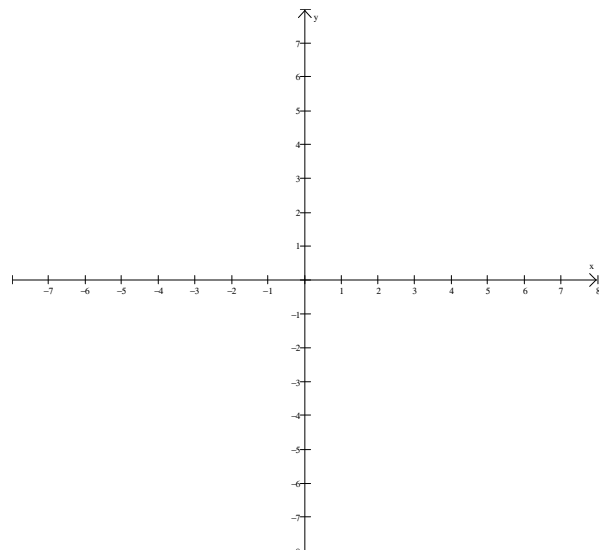
(b) When will the ball hit the ground?

15. Simplify $\frac{1+\sin \theta}{\cos \theta} + \frac{\cos \theta}{1+\sin \theta}$.

16. If the side of a cube doubles, how does the volume of the cube change?

17. Expand $\log \frac{x^3y^4}{\sqrt{z}}$ using the laws of logarithms.

18. Sketch the graph of $y = e^x$.



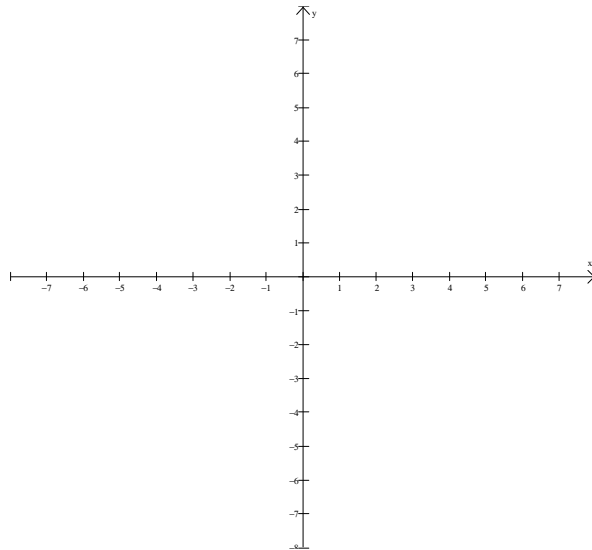
19. Solve $|2x+1| > 3$ and write your solution in **interval notation**.

20. Evaluate.

(a) $\tan^{-1}(1) = \underline{\hspace{2cm}}$

(b) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \underline{\hspace{2cm}}$

21. Graph $f(x) = \begin{cases} x^2 + 2 & \text{if } x < -1 \\ -2x + 3 & \text{if } x \geq -1 \end{cases}$



22. Given $f(x) = 2x - 1$ and $g(x) = \frac{x}{x+2}$, evaluate the following.

(a) $f(a+h)$

(b) $g(-3x)$

(c) $(g \circ f)(x)$

23. Solve $2x - 5 \leq 4$ and write your solution in **interval notation**.

24. Given $\cos \theta = \frac{-7}{10}$ and θ lies in Quadrant II, evaluate $\sin \theta$ and $\tan \theta$.

25. Given the graph of $f(x)$, sketch the graph of

$f^{-1}(x)$ on the same grid.

