Solutions to Exercises in The College Mathematics Journal
Curious Consequences of a Miscopied Quadratic

Question 1: If there are 13 compatible pairs corresponding to the integer *b*, how many of these could be primitive?

Solution: By Compatible Pair Fact 3, we have

$$13 = \frac{1}{2}\left[\prod_{i=1}^{s}(2\alpha_i + 1) - 1\right]$$

from which it follows that $27 = \prod_{i=1}^{s}(2\alpha_i + 1)$. That is, we wish to write 27 as the product of positive odd integers, each greater than 1 (where the order is unimportant). The three possibilities are $27 = 27$, $27 = 3 \cdot 9$, and $27 = 3 \cdot 3 \cdot 3$. Hence, the prime factorization of *b* could contain 1, 2, or 3 distinct primes, each congruent to 1 mod 4. By Compatible Pair Fact 4, the number of primitive compatible pairs is 1, 2, or 4.

Question 2: Of these, which has the least value for *b*?

Solution: The least value for *b* with one primitive compatible pair is
$b = 5^{13} = 1220703125$.
The least value for *b* with two primitive compatible pairs is $b = 5^4 \cdot 13 = 8125$.
The least value of *b* with four primitive compatible pairs is $b = 5 \cdot 13 \cdot 17 = 1105$.

As these are the only possible numbers of primitive compatible pairs, the minimum value for *b* with 13 compatible pairs is 1105.

Question 3: Find all the compatible pairs for $b = 1105$.

Question 4: Find all Pythagorean triples with hypotenuse $b = 1105$.

Solution: We combine the solutions to Questions 3 and 4 in the chart below. Given b and c, which were obtained with a relatively short computer program, the values of k and l are produced using the equation given equations on page 3 of the article resulting from the Quadratic Formula, and these values, in turn, lead to the values of X and Y via the relationships $X = \frac{k-l}{2}$ and $Y = \frac{k+l}{2}$.

b	c	k	l	X	Y	
1105	25944	1151	1057	47	1104	primitive
1105	141636	1337	809	264	1073	primitive
1105	275184	1519	367	576	943	primitive
1105	303924	1561	73	744	817	primitive
1105	57750	1205	995	105	1100	nonprimitive
1105	299250	1555	155	700	855	nonprimitive
1105	92274	1261	923	169	1092	nonprimitive
1105	234234	1469	533	468	1001	nonprimitive
1105	145656	1343	799	272	1071	nonprimitive
1105	267036	1513	391	561	952	nonprimitive
1105	253500	1495	455	520	975	nonprimitive
1105	216750	1445	595	425	1020	nonprimitive
1105	293046	1547	221	663	884	nonprimitive

The first four in the chart are primitive. The others are derived from primitive results as follows. The nine non-primitive compatible pairs are given below together with the primitive compatible pair that generates each.

$$x^2 - 1105x \pm 57750 \text{ derived from } x^2 - 221x \pm 2310$$
$$x^2 - 1105x \pm 299250 \text{ derived from } x^2 - 221x \pm 11970$$
$$x^2 - 1105x \pm 92274 \text{ derived from } x^2 - 85x \pm 546$$
$$x^2 - 1105x \pm 234234 \text{ derived from } x^2 - 85x \pm 1386$$
$$x^2 - 1105x \pm 145656 \text{ derived from } x^2 - 65x \pm 504$$
$$x^2 - 1105x \pm 267036 \text{ derived from } x^2 - 65x \pm 924$$
$$x^2 - 1105x \pm 253500 \text{ derived from } x^2 - 17x \pm 60$$
$$x^2 - 1105x \pm 216750 \text{ derived from } x^2 - 13x \pm 30$$
$$x^2 - 1105x \pm 293046 \text{ derived from } x^2 - 5x \pm 6$$

Finally, to further illustrate the notion of these compatible pairs, the factorizations are given in the chart below.

$(x-1128)(x+23)=$	$x^2-1105x+25944$ $x^2-1105x-25944$	$=(x-1081)(x-24)$
$(x-1221)(x+116)=$	$x^2-1105x+141636$ $x^2-1105x-141636$	$=(x-957)(x-148)$
$(x-1312)(x+207)=$	$x^2-1105x+271584$ $x^2-1105x-271584$	$=(x-736)(x-369)$
$(x-1333)(x+228)=$	$x^2-1105x+303924$ $x^2-1105x-303924$	$=(x-589)(x-516)$
$(x-1155)(x+50)=$	$x^2-1105x+57750$ $x^2-1105x-57750$	$=(x-1050)(x-55)$
$(x-1330)(x+225)=$	$x^2-1105x+299250$ $x^2-1105x-299250$	$=(x-630)(x-475)$
$(x-1183)(x+78)=$	$x^2-1105x+92274$ $x^2-1105x-92274$	$=(x-1014)(x-91)$
$(x-1287)(x+182)=$	$x^2-1105x+234234$ $x^2-1105x-234234$	$=(x-819)(x-286)$
$(x-1224)(x+119)=$	$x^2-1105x+145656$ $x^2-1105x-145656$	$=(x-952)(x-153)$
$(x-1309)(x+204)=$	$x^2-1105x+267036$ $x^2-1105x-267036$	$=(x-748)(x-357)$
$(x-1309)(x+204)=$	$x^2-1105x+253500$ $x^2-1105x-253500$	$=(x-780)(x-325)$
$(x-1275)(x+170)=$	$x^2-1105x+216750$ $x^2-1105x-216750$	$=(x-850)(x-255)$
$(x-1326)(x+221)=$	$x^2-1105x+293046$ $x^2-1105x-293046$	$=(x-663)(x-442)$