

## Chapter 14: Factor Markets

So far, we've focused our attention on markets in which firms sell output to consumers. However, firms aren't always sellers in every market. As we've seen, each firm uses inputs to produce its product, and obviously firms have to buy their inputs, so they frequently act as buyers in the markets for inputs. These markets are called factor markets because they are the markets where the factors of production (labor, capital, raw materials, land, etc.) are bought and sold, with demand in these industries being driven by firms.

In this chapter, we'll consider how competitive factor markets operate. This means that we'll need to derive the firm's demand for its inputs, aggregate these firm-level demand functions into a market demand curve, and see what we can say about the resulting market equilibrium.

Like any other market, a factor market is competitive if there are a large number of buyers and sellers, the input is homogenous, etc. Keep in mind that many input markets aren't perfectly competitive, but we'll restrict our attention to competitive markets just to keep things simple.

### Demand for an Input

Finding the firm's demand for a particular input is more straightforward than calculating a consumer's demand for a good. In the case of a consumer, demand depends on that individual consumer's utility function, which depends on that consumer's idiosyncratic preferences. In the case of a firm, however, the demand for an input is a so-called derived demand, in the sense that it is derived from the firm's desired output level and the market price of the firm's output and inputs. Given what we already know about how firms function in their respective output markets, this means that we can specify their factor demands with much more precision than what we did with consumers earlier in the course.

First, let's look at the mathematics of the firm's situation. Let's assume that the firm is in a perfectly competitive output market, so that it earns the market price  $P$  on every unit of output that it sells. Let's also assume for now that the firm has only one variable input; let  $x$  denote this input, and let  $w$  be the per-unit price of the input. The firm's production function is therefore  $q = f(x)$ , and its total cost is  $FC - wx$ , where  $FC$  is any fixed cost the firm faces not involving the variable input  $x$ .

We can write the firm's profit function as follows:

$$\pi(x) = P f(x) - FC - w x$$

Note that the first term in this function is the firm's total revenue ( $Pq$ ) and the last two terms make up the firm's total cost. This expression is a little different than what we've seen in earlier chapters, but that's only because we're writing this function in terms of the firm's input ( $x$ ) instead of its output ( $q$ ). Otherwise, it's the same.

As usual, the firm wants to select the value of its choice variable (in this case,  $x$ ) to maximize profit. We can do this by taking the derivative of the profit function with respect to  $x$ , setting the resulting derivative equal to zero, and solving for  $x$ .

$$\pi'(x) = P f'(x) - w = 0$$

$$\Rightarrow P f'(x) = w$$

This condition characterizes the firm's demand for the variable input  $x$ . In principle, we could solve this expression for  $x$ , which would be a function of the prices  $x$  and the firm's output.

Notice that  $f'(x)$  is the marginal product of input  $x$  ( $MP_x$ ). The expression  $P f'(x)$  is called the marginal revenue product of  $x$  ( $MRP_x$ ). It shows the additional revenue the firm earns when it purchases and uses one extra unit of input  $x$ . Specifically, that extra unit of  $x$  generates  $MP_x$  additional units of output, and each of those units of output sells for  $P$ , so the additional revenue from that extra unit of  $x$  must be  $(MP_x)(P)$ . This is always the case for any input, as long as the firm is in a perfectly competitive market so that its MR is always  $P$ . Obviously this would be different and a bit more complicated if the firm had market power, because in that case MR would not be a constant. Still, what we've found is that the firm will choose its use of an input so that

$$MRP = w$$

This condition has a fairly simple intuitive explanation, similar to others that we've encountered earlier. Suppose that this condition didn't hold. If  $MRP > w$  at the firm's current level of  $x$ , that means that if the firm buys one more unit of  $x$ , the extra revenue that it earns would more than cover the cost of the extra unit of  $x$ . Of course, the firm can therefore increase its profit by using more  $x$ , which implies that it isn't maximizing profit right now. Similarly, if  $MRP < w$ , that means that last unit of  $x$  didn't generate enough additional revenue to pay for itself. The last unit of  $x$  actually decreased the firm's profit, which is also clearly inconsistent with profit maximization. In order for profits to be maximized, the firm needs to keep using  $x$  up to the point that the marginal benefit from doing so ( $MRP$ ) equals the marginal cost ( $w$ ).

Since the firm will buy an input up to the point that the input's price ( $w$ ) equals its  $MRP$ , this means that the  $MRP$  is the firm's derived demand for that input.

Let's look at a quick example of how to calculate the firm's demand for an input if we have an actual production function to work with:

e.g. Suppose the firm has the production function  $q = K^{1/3}L^{1/3}$  and that the level of capital is fixed for now, so the firm only needs to choose its level of labor,  $L$ . Following the procedure described above, we can write:

$$\pi = P K^{1/3}L^{1/3} - FC - wL$$

$$d\pi/dL = (1/3)PK^{1/3}L^{-2/3} - w = 0$$

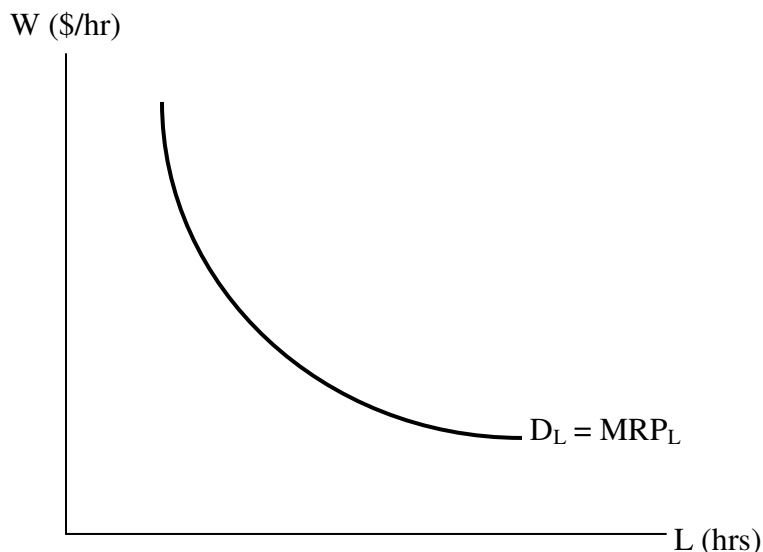
$$wL^{2/3} = (1/3)PK^{1/3}$$

$$L = P^{3/2}K^2(1/w)^{3/2}$$

This simplifies to  $L = \left(\frac{P}{3w}\right)^{3/2} K^2$ , which is this firm's demand curve for labor.

This example illustrates several properties of the firm's demand for an input. (We'll follow the example by using "labor" instead of the more nebulous "input x"):

1) Input demands slope down. Notice that in the above demand for labor, an increase in  $w$  leads to a decrease in the quantity of labor demanded. This shouldn't be surprising. On one hand, as  $w$  goes up, the firm's marginal cost is rising so it doesn't want to produce as much output, which means it doesn't need to hire as much labor. Also, remember that the firm is hiring labor up to the point that  $MRP_L = w$ . But we've seen that  $MRP_L = (P)(MP_L)$ , and we know that  $MP_L$  is decreasing in  $L$  thanks to the law of diminishing marginal returns. Hence, our demand for labor (or any other variable input) will look like this:



Notice that this matches our finding that the firm chooses  $L$  such that  $w = MRP_L$ .

2)  $\uparrow P \Rightarrow \uparrow$  input demand. An increase in the price of the firm's output will increase any input's MRP and thus increases the firm's demand for that input.

3) Anything that increases an input's MP will increase the demand for that input. This could be something like a change in technology that makes an input more productive. It could be a change in managerial systems that allows the firm to squeeze more productivity out of its inputs. Or it could be an increase in some other input that makes other inputs more productive. For example, an increase in a firm's capital will make labor more productive because each worker will have more capital to work with. This increases  $MP_L$  and therefore increases the demand for labor. (You can see this in the worked-out example above).

### When More Than One Input is Variable

In our previous example, we assumed that there was only one variable input. If there are multiple variable inputs, then things get a bit more complicated.

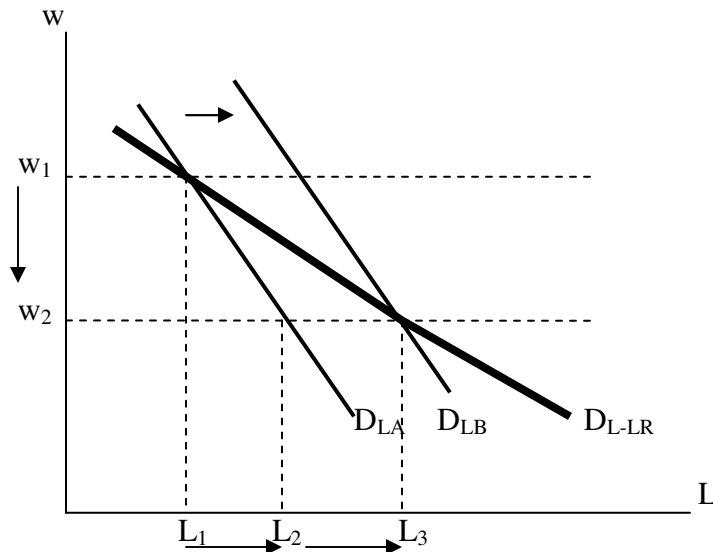
Notice that (3) above implies that a change in the price of one input will affect demand for other inputs. Suppose that capital is variable (long run) and its price rises. This will cause the firm to use less capital since the demand for capital slopes down. But we've just seen that if the firm uses less capital, the marginal product of labor will drop, so the firm will want to use less labor as well. ( $\uparrow r \Rightarrow \downarrow K \Rightarrow \downarrow MP_L \Rightarrow \downarrow D_L$ ). By similar logic, we can see that an increase in  $w$  will lead to a decrease in the demand for capital. In terms of demand, inputs are complements in the sense that an increase in one input price will decrease the demand for other inputs.

What does this tell us about a firm's long-run demand for an input compared to its short-run demand? Consider what happens to the demand for labor when  $w$  falls:

Short-run: The firm moves down along the  $D_L$  curve, buying more labor.

Long-run: In addition, the firm buys more capital (since  $K$  and  $L$  are complements). This increases  $MP_L$  and thus causes the firm to buy even more additional labor than it did in the short run.

We can see what this implies on a diagram:



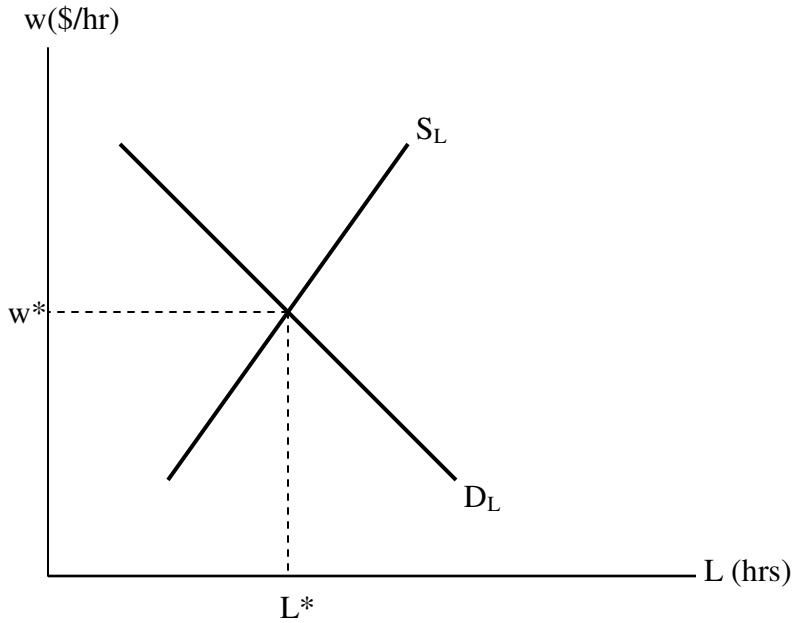
Initially, suppose  $w = w_1$  and the firm's demand for labor is  $D_{LA}$ . Then  $w$  falls to  $w_2$ . In the short run, the firm's labor increases from  $L_1$  to  $L_2$ . In the long run, though, the firm will want to buy more capital since the marginal product of capital is now higher. When it buys more capital,  $MP_L$  increases, which increases the short-run demand for labor to  $D_{LB}$ , and this increases the firm's use of labor even further to  $L_3$ . So in the short run, a decrease in  $w$  increased  $L$  from  $L_1$  to  $L_2$ , but in the long run that same decrease in  $w$  increased  $L$  all the way to  $L_3$ . This shows that the firm's demand for labor (or any other input) is more elastic in the long run than it is in the short run.

### Equilibrium in Factor Markets

In order to find an equilibrium in a factor market, we need to aggregate all the firm-level input demand curves to get market input demand. This is the exact same procedure of horizontal summation that we used in Chapter 4 to go from individual consumer demand to market demand. We also did something similar in Chapter 9 when we added up individual firms' supply curves to get market supply. Since you've seen this a couple of times before, I'll skip the derivation this time and just point out that the market demand for labor (for example), will have the same properties as the individual firm's demand for labor that we've just described. E.g. Downward-sloping, related to  $P$  and  $MP_L$ , flatter in the long run than the short run, a complement to other inputs, etc.

Now we need a supply curve. For the sake of simplicity, we will simply assume that industry supply has its normal upward slope. As the textbook points out, this is probably not actually true in the special case of the market for labor, but we will overlook this point to keep the discussion as general as possible.

We're now in a position to find an equilibrium in a factor market.

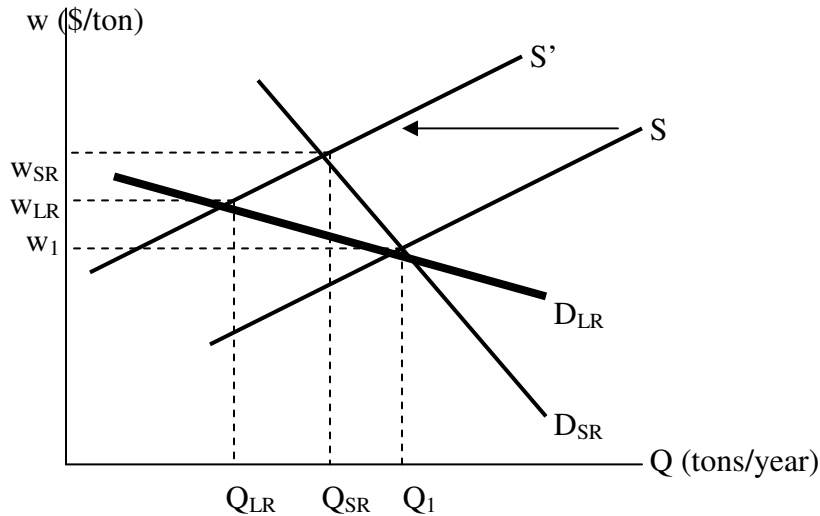


As usual, the equilibrium price of labor is the wage that equates quantity demanded with quantity supplied. If this market is competitive, then this wage and the resulting quantity ( $L$ ) are both efficient in the sense we've described before.

Our understanding of demand allows us to make comparative static predictions about factor markets as well.

e.g.  $\uparrow P \Rightarrow \uparrow D_L \Rightarrow \uparrow L$  and  $\uparrow w$

e.g. Suppose the market for steel is in equilibrium, and then several steel producers exit the industry, reducing supply. Remember that the demand for steel (like any other input) will be more elastic in the long run than it is in the short run, so our graph might look like this:



We start with an equilibrium price of  $w_1$  and an equilibrium quantity of  $Q_1$ . A decrease in supply is going to cause an increase in price and decrease in quantity, but we can also see that price will rise by more in the short run than it will in the long run. Likewise, there will be a greater decrease in quantity in the long run than in the short run.